Qualitative Properties and Regularity Theory for Nondivergence-Type Elliptic PDEs

Academic Year: 2026–2027

Instructor: Diego Ribeiro Moreira

Member of the proposing board: Fausto Ferrari

Lecture hours: 30 h

Teaching period: February to April

Scientific sector(s): Mathematical Analysis, PDEs

Course type: Advanced

Assessment method:

Upon completion of the course, students will be provided with a set of exercises related to the topics covered. Final grades of A, B, C, or D will be awarded based on student performance.

Course Abstract

This course presents the qualitative properties and regularity theory underlying elliptic partial differential equations in nondivergence form. Starting from fundamental properties of harmonic functions, the course develops a priori estimates and Schauder theory for the Poisson equation, introduces the Aleksandrov–Bakelman–Pucci maximum principle and its applications, and covers the Krylov–Safonov theory. The final part of the course is devoted to higher regularity results for nonlinear elliptic equations, culminating in the Evans–Krylov $C^{2,\alpha}$ theorem.

Course Program

• Fundamental properties of harmonic functions: gradient estimates, maximum principles, Lp–L∞ bounds, and Harnack inequalities

• The Poisson equation: a priori L∞ estimates and Schauder theory for the Laplace operator; extensions to nondivergence-form operators

• Aleksandrov–Bakelman–Pucci maximum principle and applications, including isoperimetric inequalities and symmetry results via the maximum principle and the moving planes method

• Krylov–Safonov theory: Lp–L∞ estimates, weak Harnack inequality, and Hölder regularity

• $C^{1,\alpha}$ regularity theory for nondivergence-type elliptic equations (Caffarelli–Trudinger theory)

• Evans–Krylov $C^{2,\alpha}$ theorem, with emphasis on Caffarelli’s proof